

IRRATIONAL EQUATIONS

- Irrational equations are equations where surds are involved.
- Solving irrational equations may involve rearranging before squaring each side.
- Once you have solutions, substitute the solution back into the original equation to check that the solutions hold. Some solutions may be eliminated in this step.

Example 1: Solve $\sqrt{(5x + 14)} = x$.

$$\sqrt{(5x + 14)} = x$$

$$(5x + 14) = x^2$$

squaring both sides

$$x^2 - 5x - 14 = 0$$

rearrange to make a quadratic equation

$$(x - 7)(x + 2) = 0$$

solve the quadratic

$$x = 7 \text{ or } x = -2$$

$$\text{For } x = 7, \sqrt{(35 + 14)} = 7$$

test the solution

$$\text{For } x = -2, \sqrt{(-10 + 14)} = 2$$

solution does not hold

Solution is $x = 7$.

Example 2: Solve $x - \sqrt{(x + 8)} - 4 = 0$.

$$x - \sqrt{(x + 8)} - 4 = 0$$

$$\sqrt{(x + 8)} = x - 4$$

$$(x + 8) = (x - 4)^2$$

squaring both sides

$$x + 8 = x^2 - 8x + 16$$

$$x^2 - 9x + 8 = 0$$

rearrange to make a quadratic equation

$$(x - 8)(x - 1) = 0$$

solve the quadratic

$$x = 8 \text{ or } x = 1$$

$$\text{For } x = 8: 8 - \sqrt{(8 + 8)} - 4 = 0$$

test the solution

$$\text{For } x = 1: 1 - \sqrt{(1 + 8)} - 4 = -6$$

solution does not hold

Solution is $x = 8$.

Practice Questions: Solve

$$1) 2 - x + \sqrt{2x - 1} = 0$$

$$2) x - 2 = 3\sqrt{x - 2}$$

$$3) \sqrt{2x - 1} = 1 + \sqrt{x - 1}$$

$$4) \sqrt{x + 1} - 2 = \sqrt{x - 7}$$

$$5) \sqrt{t - 1} + \sqrt{t + 4} = 5$$

$$6) \sqrt{x + 4} - \sqrt{2} = \sqrt{x - 6}$$

Solutions:

1) $x = 5$ or $x = 1$, but $x = 1$ does not hold, so solution is $x = 5$.

2) $x = 11$ or $x = 2$.

3) $x = 3$ or $x = 1$, but $x = 3$ does not hold, so solution is $x = 1$.

4) $x = 8$

5) $t = 5$

6) $x = 14$